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Aggregation with price variation and heterogeneity across consumers

Abstract:
In the aggregation literature, prices and price and income derivatives are often assumed not to vary across consumers. These assumptions may not be fulfilled: prices often vary and consumers are heterogeneous in the way they respond to price and income changes. In this paper we develop and illustrate a framework for consistent aggregation over consumers within an Almost Ideal Demand System (AIDS) model where prices, total expenditure and the response to these variables vary across consumers. We show how the frameworks previously discussed in the literature on aggregation of an AIDS model may be written as a special case within this general framework. The method may easily be adapted to any functional form.

Keywords: Aggregation, Consumer demand, Heterogeneity

JEL classification: D1

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1. Introduction

In order to achieve political goals, policy instruments are used to change *individual* behaviour. The effectiveness of these instruments depends, however, on the *aggregate* response of all consumers. Unless consumers are, to some degree, homogeneous in their demand, we may experience an aggregation problem if we wish to give the macro function a behavioural interpretation (see e.g., Mas-Colell et al., 1995). The reason for the aggregation problems is that the parameters of the macro function include structural as well as behavioural components (Stoker, 1993). The difference between the micro and macro parameters caused by these structural components creates a disaggregation bias if we use micro parameters to predict the aggregate demand response as if they were macro parameters (Denton and Mountain, 2001 and 2004). These aggregation and disaggregation biases create the need for theoretically consistent framework for calculating macro demand using micro demand.

Several studies discuss the differences between the properties of the micro and macro demand function.[1] Even if these studies do not necessarily aim at creating a framework for calculating macro demand response, it is possible to use several of these approaches to find theoretically consistent macro demand using micro data. They are, however, often specific to a functional form: many are based on (QU)AIDS models (see e.g., Deaton and Muellbauer, 1980, Blundell et al., 1993, Denton and Mountain, 2001 and 2004). Furthermore, in most of this literature, it is assumed that prices and price responses do not vary across consumers.[2] With some exceptions (for instance, Blundell et al., 1993), the income response is also assumed equal across consumers. In reality, however, prices do vary across consumers and there is heterogeneity in consumers' response to price and income changes. Thus, even if these approaches account for several sources of heterogeneity across consumers, we may still experience a disaggregation bias when using these methods to obtain an estimate of the macro demand response of a policy instrument.

The novelty of this paper is to develop a general framework for calculating macro parameters using micro information in the case where income, prices and the price and income derivatives are allowed to vary across consumers. We do this by separating the behavioural and structural components of the macro function. Heterogeneity in behaviour is modelled similarly to Blundell et al. (1993). However,

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[1] Forni and Brighi (1991) summarise the theoretical literature on aggregation. Blundell and Stoker (2005) give a survey of studies dealing with aggregation in applied research.

[2] The exception is Lau and Wu's work on exact aggregation where prices may vary across consumers (Lau and Wu, 1996). However, and as noted by Forni and Brighi (1991), exact aggregation does not secure the behavioural interpretation of the macro demand function necessary for policy analysis.

Blundell et al. (1993) only models heterogeneity in the constant term and income parameter with dummy variables, whereas we allow all demand derivatives to vary with both discrete and continuous consumer characteristics. We illustrate our approach using an AIDS model. This framework may, however, be easily applied to other functional forms. In this way, the framework allows us to use the functional form that best describes data, without making behavioural assumptions at the micro level to enable aggregation.

2. Aggregation of heterogeneous consumers

We know from the aggregation literature that when consumers are heterogeneous, changes in, for example, the income distribution, may affect macro demand through both behavioural effects and structural changes. Thus, the objective of our approach is to write the macro function in a theoretically consistent way, where we separate the behavioural and structural components. To do this, we need to write the macro parameters as a function of: 1) the micro parameters describing heterogeneity across consumers (behavioural components), 2) macro variables (prices, income and consumer characteristics important for describing the heterogeneity in demand) and 3) parameters describing the distribution of all variables (hereafter called aggregation factors). The macro variables for consumer characteristics describing heterogeneity (number of household members, age structure, education, etc.) and the aggregation factors (that is, items 2 and 3) are the structural components of the macro parameters, whereas the micro parameters are the behavioural components. A change in prices or income may have both structural and behavioural effects on macro demand. How the structural components enter the macro function depends on the specification of the micro functions.

We now show how to separate the structural and behavioural components within an AIDS model where income, prices and price and income derivatives are allowed to vary across consumers. We assume that all the properties of the standard AIDS system are fulfilled for each consumer.[3] However, we assume that heterogeneity exists across consumers; i.e., we assume that there exist $h=1, \ldots, H$ independent AIDS demand systems. When prices, total expenditure and all parameters are allowed to vary across consumers, the budget share of good i for consumer h, w_i^h, is given by:

(1) $$w_i^h = \alpha_i^h + \sum_j \gamma_{ij}^h \log\left(p_j^h\right) + \beta_i^h \log\left(\breve{x}^h\right),$$

[3] See Deaton and Muellbauer (1980) for the initial description of the AIDS demand system.

where p_j^h is the price of good j for consumer h ($j=1, \ldots, J^h$), J^h is the number of goods the consumer has the opportunity to consume and α_i^h, γ_{ij}^h and β_i^h are individual consumer parameters. The total real expenditure for consumer h is given by $\breve{x}^h = x^h / P^h$, where x^h is total expenditure, and the price index (P^h) is defined implicitly by $\log(P^h) = \alpha_0^h + \sum_l \alpha_l^h \log(p_l^h) + \tfrac{1}{2}\sum_l \sum_j \gamma_{jl}^h \log(p_l^h)\log(p_j^h)$,

$l = 1, \ldots, J^h$.

We model heterogeneity by letting all micro parameters depend on sociodemographic variables:

$$
\begin{aligned}
\alpha_i^h &= \alpha_0^i + \alpha_1^i D^h + \alpha_2^i K^h, \\
\gamma_{ij}^h &= \gamma_0^{ij} + \gamma_1^{ij} D^h + \gamma_2^{ij} K^h \qquad , \\
\beta_i^h &= \beta_0^i + \beta_1^i D^h + \beta_2^i K^h,
\end{aligned} \tag{2}
$$

where D indicates a dummy variable and K is a continuous variable. For simplicity, and without loss of generality, we assume one discrete and one continuous variable, and that the same variables affect all parameters.

The macro budget share, measured in terms of the arithmetic mean values of all variables, is defined as:[4]

$$
w_i = \frac{\overline{q_i}\,\overline{p_i}}{\overline{x}}. \tag{3}
$$

Since prices and total expenditure, as well as consumption, vary across consumers, the macro and mean budget share will differ, such that $w_i \neq \overline{w_i}$.

To find the expression for the macro budget share in Equation (3), we calculate mean consumption given the demand structure in Equations (1) and (2). Inserting the parameters in Equation (2) into the

[4] Hereafter, 'bar' denotes the arithmetic mean, e.g., $\overline{q_i} = \frac{1}{H}\sum_h q_i^h$.

expression for mean consumption, $\overline{q}_i = \frac{1}{H}\sum_h \left\{ \left[\alpha_i^h + \sum_j \gamma_{ij}^h \log\left(p_j^h\right) + \beta_i^h \log\left(\breve{x}^h\right) \right] \frac{x^h}{p_i^h} \right\}$, and rearranging, we have:

$$
\begin{aligned}
\overline{q}_i \quad = \quad & \left[\alpha_0^i S_0^i + \alpha_1^i \overline{D} S_D^i + \alpha_2^i \overline{K} S_K^i\right] \frac{\overline{x}}{\overline{p}_i} \\
& + \sum_j \left[\gamma_0^{ij} S_{0,p_j}^i + \gamma_1^{ij} \overline{D} S_{D,p_j}^i + \gamma_2^{ij} \overline{K} S_{K,p_j}^i\right] \log\left(\overline{p_j}\right) \frac{\overline{x}}{\overline{p}_i} \qquad (4) \\
& + \left[\beta_0^i S_{0,x}^i + \beta_1^i \overline{D} S_{D,x}^i + \beta_2^i \overline{K} S_{K,x}^i\right] \log\left(\overline{\breve{x}}\right) \frac{\overline{x}}{\overline{p}_i}
\end{aligned}
$$

We can see from Equation (4) that the macro demand function is written as a function of the micro parameters, arithmetic mean values for total expenditure, total real expenditure, prices and characteristics and the aggregation factors, defined by:

(i) $$S_0^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h},$$

(ii) $$S_D^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{D^h}{\overline{D}},$$

(iii) $$S_K^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{K^h}{\overline{K}},$$

(iv) $$S_{0,p_j}^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{\log\left(p_j^h\right)}{\log\left(\overline{p_j}\right)},$$

(v) $$S_{D,p_j}^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{D^h}{\overline{D}} \frac{\log\left(p_j^h\right)}{\log\left(\overline{p_j}\right)},$$

(vi) $$S_{K,p_j}^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{K^h}{\overline{K}} \frac{\log\left(p_j^h\right)}{\log\left(\overline{p_j}\right)},$$

(vii) $$S_{0,x}^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{\log\left(\breve{x}^h\right)}{\log\left(\overline{\breve{x}}\right)},$$

(viii) $$S_{D,x}^i = \frac{1}{H}\sum_h \frac{x^h}{\overline{x}} \frac{\overline{p_i}}{p_i^h} \frac{D^h}{\overline{D}} \frac{\log\left(\breve{x}^h\right)}{\log\left(\overline{\breve{x}}\right)}$$ and

(ix) $$S_{K,x}^{i} = \frac{1}{H}\sum_{h}\frac{x^{h}}{\overline{x}}\frac{\overline{p_i}}{p_i^{h}}\frac{K^{h}}{\overline{K}}\frac{\log\left(\breve{x}^{h}\right)}{\log\left(\overline{\breve{x}}\right)}.$$

These aggregation factors depend on the distribution of prices, total expenditure, total real expenditure and consumer characteristics. Inserting the expression for mean consumption in Equation (4) into the expression for the macro budget share in Equation (3), we can write the macro budget share as a function of the macro parameters, mean prices and mean total real expenditure:

(5) $$w_i = \tilde{\alpha}_i + \sum_{j}\tilde{\gamma}_{ij}\log\left(\overline{p_j}\right) + \tilde{\beta}_i \log\left(\overline{\breve{x}}\right),$$

where the relationship between the micro and macro parameters corrected for structural components are given by (see Appendix A for details):

(x) $$\tilde{\alpha}_i = \alpha_0^i S_0^i + \alpha_1^i \overline{D} S_D^i + \alpha_2^i \overline{K} S_K^i,$$

(xi) $$\tilde{\gamma}_{ij} = \gamma_0^{ij} S_{0,p_j}^i + \gamma_1^{ij} \overline{D} S_{D,p_j}^i + \gamma_2^{ij} \overline{K} S_{K,p_j}^i \quad \text{and}$$

(xii) $$\tilde{\beta}_i = \beta_0^i S_{0,x}^i + \beta_1^i \overline{D} S_{D,x}^i + \beta_2^i \overline{K} S_{K,x}^i.$$

As shown in Equations (x)-(xii), the macro parameters ($\tilde{\alpha}_i$, $\tilde{\gamma}_{ij}$ and $\tilde{\beta}_i$) depend on the micro parameters (α_0^i, α_1^i, α_2^i, γ_0^{ij}, γ_1^{ij}, γ_2^{ij}, β_0^i, β_1^i and β_2^i), the mean values of the variables describing heterogeneity ($\overline{K}$ and $\overline{D}$) and the aggregation factors (all of the *S*'s). The mean values of the consumer characteristics capture the structural effects on the macro parameter resulting from the fact that consumers react differently to price and income changes. The aggregation factors capture the structural effects of changes in the distribution of variables. The aggregation factors depend both on the distribution in the population, and in different subsamples of consumers. This can be seen by rewriting the aggregation factors including the dummy variable *D*, e.g. S_D^i may be written as $S_D^i = \frac{1}{H_D}\sum_{h\in M_D}\frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h}$, where H_D is the number of consumers and M_D is the set of consumers with the characteristic *D* (see Appendix A for detailed calculations). We can see from this equation that S_D^i equals S_0^i for households within the set M_D.

Equations (x)-(xii) show under what assumptions the macro demand function has a behavioural interpretation; that is, when there are no structural effects so the micro and macro parameters are equal. The macro parameters for the constant and price term will equal the micro parameters only if all consumers face equal prices ($p_i^h = p_i$ and $p_j^h = p_j$) and either all consumers are equal ($K^h = K$ and $D^h = D$) or have an equal demand structure ($\alpha_1^i = \alpha_2^i = \gamma_1^{ij} = \gamma_2^{ij}$ =0). Inserting the assumptions that all consumers face equal prices and have equal sociodemographic characteristics into the aggregation factors, we find that most equal one ($S_0^i = S_D^i = S_K^i = S_{0,p_j}^i = S_{D,p_j}^i = S_{K,p_j}^i$ = 1). However, as long as total real expenditure varies across consumers, the aggregation factors including total real expenditure are $S_{0,x}^i = S_{D,x}^i = S_{K,x}^i = \dfrac{1}{H}\sum_h \dfrac{x^h}{\bar{x}} \dfrac{\log(\breve{x}^h)}{\log(\bar{\breve{x}})}$. This implies that the macro parameter for total real expenditure ($\tilde{\beta}_i$) always differs from the micro parameter, even if all prices are equal and there is no heterogeneity in demand ($\beta_1^i = \beta_2^i$ =0). Thus, as long as prices, total expenditure or the micro parameters vary across consumers, structural effects on macro demand in an AIDS model will exist.

3. Comparison with the literature

We now show how the AIDS model previously discussed in the literature may be written as a special case of our general framework in Equation (5). We focus on two approaches, Deaton and Muellbauer (1980) and Denton and Mountain (2001).[5]

In Deaton and Muellbauer (1980), consumers are assumed to be identical with the exception of variation in income and a parameter k^h, which is introduced in the micro budget share function to correct the total real expenditure effects for variations in consumer characteristics. The micro budget share function in Deaton and Muellbauer (1980) is given by:

(6) $$w_i^h = \alpha^i + \sum_j \gamma^{ij} \log(p_j) + \beta^i \log\left(\frac{x^h}{k^h P}\right),$$

where P is the price index under the assumption that prices and parameters are equal across consumers ($P^h = P$). Deaton and Muellbauer's macro budget share function is given by:

---

[5] It is also possible to apply our approach to a QUAIDS model, and show that Denton and Mountain (2004) and Blundell et al. (1993) are special cases of this extension. For a discussion of the QUAIDS model, see Blundell et al. (1993).

$$(7) \qquad W_i = \frac{\sum_h p_i q_i^h}{\sum_h x^h} = \alpha^i + \sum_j \gamma^{ij} \log(p_j) + \beta^i \log\left(\frac{\bar{x}}{kP}\right),$$

where k^h is aggregated to what Deaton and Muellbauer call a taste index, k. In their paper, Deaton and Muellbauer focus on the case where k^h=1 for all h. In this case, they write the index k as Theil's entropy measure (Z) divided by the number of households ($k=Z/H$), where Z is implicitly given by $\log(Z) = -\sum_h \frac{x^h}{X} \log\left(\frac{x^h}{X}\right)$ and X is aggregate total expenditure ($X = \sum_h x^h$).

To show the relationship between our macro function in Equation (5) and that of Deaton and Muellbauer in Equation (7), we assume that all consumers are homogeneous with respect to changes in prices and total expenditure; i.e., the effects of D and K equal zero in Equation (2). Further, we assume that all consumers face the same prices, i.e., $p_j^h = p_j \ \forall \ j, \ h$. In this case, our micro function is given by:

$$(8) \qquad w_i^h = \alpha_0^i + \sum_j \gamma_0^{ij} \log(p_j) + \beta_0^i \log(\breve{x}^h) .$$

We can see from Equations (6) and (8) that when k^h=1 for all h, our micro function is equal to Deaton and Muellbauer's micro function. Under Deaton and Muellbauer's assumptions, $S_0^i = S_{0,p_j}^i$ =1, $\tilde{\alpha}_i = \alpha_0^i$, $\tilde{\gamma}_{ij} = \gamma_0^{ij}$ and $\tilde{\beta}_i = \beta_0^i S_{0,x}^i$, where $S_{0,x}^i = \frac{1}{H} \sum_h \frac{x^h}{\bar{x}} \frac{\log(\breve{x}^h)}{\log(\bar{\breve{x}})} = S_{0,x}$ is a measure of total expenditure variation. In this case, our macro budget share function is given by:

$$(9) \qquad w_i = \alpha_0^i + \sum_j \gamma_0^{ij} \log(p_j) + \beta_0^i S_{0,x} \log(\bar{\breve{x}}) .$$

We may write the relationship between the taste index k and the aggregation factor $S_{0,x}$ as $S_{0,x} = 1 - \frac{\log(k)}{\log(\bar{\breve{x}})}$ (see Appendix B for detailed calculations).

From this, we may show that Deaton and Muellbauer's model is nested within our macro budget share function. This is under the assumptions that all consumers face the same prices, all consumers are homogeneous with respect to changes in prices and total real expenditure, and k^h=1 for all h. Detailed calculations of the comparison with Deaton and Muellbauer is given in Appendix B.

In Denton and Mountain (2001), the specification of the macro budget share function is given by:

$$(10) \qquad \omega_i = \alpha_0^i + \sum_j \gamma_0^{ij} \log(p_j) + \beta_0^i \sum_h \frac{x^h}{X} \log\left(\frac{x^h}{P}\right),$$

which is identical to our macro budget share function in Equation (9), in which we assume that all consumers are homogenous with respect to changes in prices and total expenditure and face the same prices (see Appendix B for detailed calculations).

4. Concluding remarks

Most applied analyses discussing aggregation and disaggregation biases employ standard demand systems, such as (Q)LES or (QU)AIDS, with no heterogeneity in parameters and no price variation across consumers. One reason is that the aggregated properties of these systems are well known. Whether the functional form describes the data well is an empirical question. Assuming that consumer heterogeneity and price variation across consumers is nonexistent does not reduce the aggregation or disaggregation biases in the case where consumers are in fact heterogeneous and prices vary: it only conceals them. For example, Denton and Mountain (2001 and 2004) calculate the aggregation/disaggregation biases under these assumptions, and conclude that the biases are small. The question is whether this conclusion will hold using a more general framework allowing for greater heterogeneity across consumers.

We agree with the conclusion of Blundell and Stoker (2005, p. 385) that one needs to account for aggregation problems explicitly in empirical analyses that aim to analyse the effect on macro consumption of price and income: "The practice of ignoring or closeting aggregation problems as "just too hard" is no longer appropriate". The point we want to stress is that when considering aggregation problems, we need to allow for price variation and heterogeneity in price and income response, as well as income variation. Otherwise, we may ignore potentially important sources of aggregation and disaggregation bias.

References

Blundell, R., P. Pashardes and G. Weber, 1993, What do we learn about consumer demand patterns from micro data?, American Economic Review 83, 570-597.

Blundell, R. and T.M. Stoker, 2005, Heterogeneity and aggregation, Journal of Economic Literature 43, 347-391.

Deaton, A. and J. Muellbauer, 1980, An almost ideal demand system, American Economic Review 70, 312-326.

Denton F.T. and D.C. Mountain, 2001, Income distribution and aggregation/disaggregation biases in the measurement of consumer demand elasticities, Economics Letters 73, 21-28.

Denton F.T. and D.C. Mountain, 2004, Aggregation effects on price and expenditure elasticities in a quadratic almost ideal demand system, Canadian Journal of Economics 37, 613-628.

Forni, M. and L. Brighi, 1991, Aggregation across agents in demand systems, Ricerche Economiche 45, 79-114.

Lau, L.J. and H.-M. Wu, 1996, Exact aggregation under summability and homogeneity with individually variable prices, Economics Letters 50, 329-335.

Mas-Colell, A., M.D. Whinston and J.R. Green, 1995, Microeconomic theory (Oxford University Press, New York).

Stoker, T., 1993, Empirical approaches to the problem of aggregation over individuals, Journal of Economic Literature 31, 1827-1874.

Calculation of macro parameters

In this appendix, we show how to aggregate the micro budget share function in Equation (1) to obtain the aggregate demand function in Equation (4).

First, we aggregate the constant term in the micro function, resulting in the term including the macro constant parameter ($\tilde{\alpha}_i$):

(A1)

$$
\begin{aligned}
\frac{1}{H}\sum_h \alpha_i^h \frac{x^h}{p_i^h} &= \frac{1}{H}\sum_h\left[\left(\alpha_0^i + \alpha_1^i D^h + \alpha_2^i K^h\right)\frac{x^h}{p_i^h}\right] \\
&= \left(\frac{\alpha_0^i}{H}\sum_h \frac{x^h}{p_i^h} + \frac{\alpha_1^i}{H}\sum_h D^h \frac{x^h}{p_i^h} + \frac{\alpha_2^i}{H}\sum_h K^h \frac{x^h}{p_i^h}\right)\frac{\overline{p_i}}{\overline{p_i}}\frac{\overline{x}}{\overline{x}} \\
&= \left[\frac{\alpha_0^i}{H}\sum_h \frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h} + \frac{\alpha_1^i}{H}\sum_h D^h \frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h} + \frac{\alpha_2^i}{H}\sum_h K^h \frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h}\right]\frac{\overline{x}}{\overline{p_i}} \\
&= \left[\alpha_0^i S_0^i + \alpha_1^i \overline{D} S_D^i + \alpha_2^i \overline{K} S_K^i\right]\frac{\overline{x}}{\overline{p_i}} \\
&= \tilde{\alpha}_i \frac{\overline{x}}{\overline{p_i}}
\end{aligned}
$$

Second, we aggregate the price effects, resulting in the term including the macro price parameter ($\tilde{\gamma}_{ij}$):

(A2)

$$
\begin{aligned}
\frac{1}{H}\sum_h\sum_j \gamma_{ij}^h \log\left(p_j^h\right)\frac{x^h}{p_i^h} &= \frac{1}{H}\sum_h\sum_j\left(\gamma_0^{ij} + \gamma_1^{ij} D^h + \gamma_2^{ij} K^h\right)\log\left(p_j^h\right)\frac{x^h}{p_i^h} \\
&= \sum_j\left[\frac{\gamma_0^{ij}}{H}\sum_h \frac{x^h \log\left(p_j^h\right)}{p_i^h} + \frac{\gamma_1^{ij}}{H}\sum_h D^h \frac{x^h \log\left(p_j^h\right)}{p_i^h} + \frac{\gamma_2^{ij}}{H}\sum_h K^h \frac{x^h \log\left(p_j^h\right)}{p_i^h}\right]\frac{\overline{x}}{\overline{x}}\frac{\overline{p_i}}{\overline{p_i}}\frac{\log\left(\overline{p_j}\right)}{\log\left(\overline{p_j}\right)} \\
&= \sum_j\left[\frac{\gamma_0^{ij}}{H}\sum_h \frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h}\frac{\log\left(p_j^h\right)}{\log\left(\overline{p_j}\right)} + \frac{\gamma_1^{ij}}{H}\sum_h D^h \frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h}\frac{\log\left(p_j^h\right)}{\log\left(\overline{p_j}\right)} + \frac{\gamma_2^{ij}}{H}\sum_h K^h \frac{x^h}{\overline{x}}\frac{\overline{p_i}}{p_i^h}\frac{\log\left(p_j^h\right)}{\log\left(\overline{p_j}\right)}\right]\frac{\overline{x}\log\left(\overline{p_j}\right)}{\overline{p_i}} \\
&= \sum_j\left[\gamma_0^{ij} S_{0,p_j}^i + \gamma_1^{ij}\overline{D} S_{D,p_j}^i + \gamma_2^{ij}\overline{K} S_{K,p_j}^i\right]\frac{\overline{x}\log\left(\overline{p_j}\right)}{\overline{p_i}} \\
&= \tilde{\gamma}_{ij}\log\left(\overline{p_j}\right)\frac{\overline{x}}{\overline{p_i}}
\end{aligned}
$$

Finally, we aggregate the total real expenditure effects, resulting in the term including the macro total real expenditure parameter ($\tilde{\beta}_i$):

(A3)
$$
\begin{aligned}
\frac{1}{H}\sum_h \beta_i^h \log\left(x^h\right)\frac{x^h}{p_i^h} &= \frac{1}{H}\sum_h \left(\beta_0^i + \beta_1^i D^h + \beta_2^i K^h\right)\log\left(\breve{x}^h\right)\frac{x^h}{p_i^h} \\
&= \left[\frac{\beta_0^i}{H}\sum_h \frac{x^h \log\left(\breve{x}^h\right)}{p_i^h} + \frac{\beta_1^i}{H}\sum_h D^h \frac{x^h \log\left(\breve{x}^h\right)}{p_i^h} + \frac{\beta_2^i}{H}\sum_h K^h \frac{x^h \log\left(\breve{x}^h\right)}{p_i^h}\right]\frac{\bar{x}}{\bar{x}}\frac{\bar{p}_i}{\bar{p}_i}\frac{\log\left(\bar{\breve{x}}\right)}{\log\left(\bar{\breve{x}}\right)} \\
&= \left[\frac{\beta_0^i}{H}\sum_h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}\frac{\log\left(\breve{x}^h\right)}{\log\left(\bar{\breve{x}}\right)} + \frac{\beta_1^i}{H}\sum_h D^h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}\frac{\log\left(\breve{x}^h\right)}{\log\left(\bar{\breve{x}}\right)} + \frac{\beta_2^i}{H}\sum_h K^h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}\frac{\log\left(\breve{x}^h\right)}{\log\left(\bar{\breve{x}}\right)}\right]\frac{\bar{x}}{\bar{p}_i}\log\left(\bar{\breve{x}}\right) \\
&= \left[\beta_0^i S_{0,x}^i + \beta_1^i \bar{D} S_{D,x}^i + \beta_2^i \bar{K} S_{K,x}^i\right]\frac{\bar{x}}{\bar{p}_i}\log\left(\bar{\breve{x}}\right) \\
&= \tilde{\beta}_i \log\left(\bar{\breve{x}}\right)\frac{\bar{x}}{\bar{p}_i}
\end{aligned}
$$

We also show that S_D^i may be written as $S_D^i = \frac{1}{H_D}\sum_{h\in M_D}\frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}$:

(A4)
$$
\begin{aligned}
S_D^i &= \frac{1}{H}\sum_h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}\frac{D^h}{\bar{D}} \\
&= \frac{1}{H}\sum_h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}\frac{D^h}{\frac{1}{H}\sum_h D^h} \\
&= \sum_h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}\frac{D^h}{H_D} \\
&= \frac{1}{H_D}\sum_h \frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}D^h \\
&= \frac{1}{H_D}\sum_{h\in M_D}\frac{x^h}{\bar{x}}\frac{\bar{p}_i}{p_i^h}
\end{aligned}
$$

Appendix B

Calculations in the comparison with the literature

To show that our approach equals that in Deaton and Muellbauer (1980) and Denton and Mountain (2001), we calculate our macro budget share function in the case where all consumers face the same prices ($p_i^h = p_i$ and $p_j^h = p_j$) and there is no consumer heterogeneity ($\alpha_1^i = \alpha_2^i = \gamma_1^{ij} = \gamma_2^{ij} = 0$), that is, under Deaton and Muellbauer's assumptions. Then we prove that Deaton and Muellbauer (1980) and Denton and Mountain (2001) are nested within our aggregated budget share function under these assumptions.

The term including the macro constant parameter ($\tilde{\alpha}_i$) is given by:

$$
\text{(B1)} \qquad
\begin{aligned}
\sum_h \alpha_i^h \frac{x^h}{p_i} &= \sum_h \left[\alpha_0^i \frac{x^h}{p_i} \right] \\
&= \left(\alpha_0^i \sum_h \frac{x^h}{p_i} \right) \frac{\bar{x}}{\bar{x}} \\
&= \left[\alpha_0^i \sum_h \frac{x^h}{X} \right] \frac{X}{p_i} \\
&= \alpha_0^i \frac{X}{p_i}
\end{aligned}
$$

The term including the macro price parameter ($\tilde{\gamma}_{ij}$) is given by:

$$
\text{(B2)} \qquad
\begin{aligned}
\sum_h \sum_j \gamma_{ij}^h \log(p_j) \frac{x^h}{p_i} &= \sum_h \sum_j \gamma_0^{ij} \log(p_j) \frac{x^h}{p_i} \\
&= \sum_j \left[\gamma_0^{ij} \sum_h \frac{x^h \log(p_j)}{p_i} \right] \frac{X}{X} \\
&= \sum_j \left[\gamma_0^{ij} \sum_h \frac{x^h}{X} \right] \frac{X \log(p_j)}{p_j} \\
&= \sum_j \gamma_0^{ij} \log(p_j) \frac{X}{p_j}
\end{aligned}
$$

The term including the macro total real expenditure parameter ($\tilde{\beta}_i$) is given by:

$$\text{(B3)}\quad \begin{aligned}\sum_h \beta_i^h \log\left(\breve{x}^h\right)\frac{x^h}{p_i} &= \sum_h \beta_0^i \log\left(\breve{x}^h\right)\frac{x^h}{p_i}\\ &=\left[\beta_0^i \sum_h \frac{x^h \log\left(\breve{x}^h\right)}{p_i}\right]\frac{X}{X}\\ &=\left[\beta_0^i \sum_h \frac{x^h \log\left(\breve{x}^h\right)}{X}\right]\frac{X}{p_i}\end{aligned}$$

This gives the following expression for the macro budget share function:

$$\text{(B4)}\quad w_i = \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \sum_h \frac{x^h \log\left(\breve{x}^h\right)}{X}\ .$$

To prove that this macro budget share function equals the one in Deaton and Muellbauer (1980), we use that $\breve{x}^h = x^h / P^h$, $P^h = P \ \forall \ h$, $\sum_h x^h = X$, $\log(Z) = -\sum_h \frac{x^h}{X}\log\left(\frac{x^h}{X}\right)$, *k=Z/H*, $X / H = \bar{x}$, and insert these into (B4):

$$\text{(B5)}\quad \begin{aligned} w_i &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \sum_h \left[\frac{x^h}{X}\log\left(x^h\right) - \frac{x^h}{X}\log(P)\right]\\ &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \left[\sum_h \frac{x^h}{X}\log\left(x^h\right) - \log(P)\sum_h \frac{x^h}{X} - \log(X) + \log(X)\right]\\ &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \left[\sum_h \frac{x^h}{X}\log\left(\frac{x^h}{X}\right) + \log\left(\frac{X}{P}\right)\right]\\ &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \left[\log\left(\frac{X}{P}\right) - \log(Z)\right]\\ &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \log\left(\frac{X}{kHP}\right)\\ &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \log\left(\frac{\bar{x}}{kP}\right) = W_i \end{aligned}$$

which equals Deaton and Muellbauer's macro budget share function (see Equation (7)). The relationship between Theil's entropy measure and our aggregation parameter $S_{0,x}$, under these assumptions, is:

$$
\begin{aligned}
S_{0,x} &= \frac{1}{H}\sum_h \frac{x^h}{\bar{x}} \frac{\log\left(\breve{x}^h\right)}{\log\left(\bar{\breve{x}}\right)} \\
&= \frac{1}{H\log\left(\bar{\breve{x}}\right)} \sum_h \frac{x^h H}{X} \log\left(\breve{x}^h\right) \\
&= \frac{1}{\log\left(\bar{\breve{x}}\right)} \sum_h \left[\frac{x^h}{X} \left(\log\left(x^h\right) - \log\left(P\right) - \log\left(X\right) + \log\left(X\right) \right) \right] \\
&= \frac{1}{\log\left(\bar{\breve{x}}\right)} \left[\log\left(\frac{X}{P}\right) + \sum_h \frac{x^h}{X} \log\left(\frac{x^h}{X}\right) \right] \\
&= \frac{1}{\log\left(\bar{\breve{x}}\right)} \left[\log\left(\frac{X}{P}\right) - \log\left(Z\right) \right] \\
&= \frac{\log\left(\frac{X}{PH}\right) - \log\left(k\right)}{\log\left(\bar{\breve{x}}\right)} \\
&= 1 - \frac{\log\left(k\right)}{\log\left(\bar{\breve{x}}\right)}
\end{aligned}
\tag{B6}
$$

The relationship between our macro budget share function (Equation (9)) and the one in Denton and Mountain (2001) (Equation (10)) is given by:

$$
\begin{aligned}
w_i &= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \, S_{0,x} \log\left(\bar{\breve{x}}\right) \\
&= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \frac{1}{H} \sum_h \frac{x^h}{\bar{x}} \frac{\log\left(\breve{x}^h\right)}{\log\left(\bar{\breve{x}}\right)} \log\left(\bar{\breve{x}}\right) \\
&= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \sum_h \frac{x^h}{X} \log\left(\breve{x}^h\right) \\
&= \alpha_0^i + \sum_j \gamma_0^{ij} \log\left(p_j\right) + \beta_0^i \sum_h \frac{x^h}{X} \log\left(\frac{x^h}{P}\right) = \omega_i
\end{aligned}
\tag{B7}
$$

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